M.Sc. - I (Mathematics) (NEP Pattern) Semester-II

Major Elective DSE-2 - Differential Geometry

P. Pages : 2 **GUG/S/25/15397**

Time : Three Hours Max. Marks : 80

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Notes : 1. Solve **all five** questions.
2. Each question carries equal marks.

UNIT – I

1. a) Prove that the first fundamental form of a surface is a positive definite quadratic form in du, dv. **8**

b) If (l', m') are the direction coefficient of a line which makes an angle $\frac{\pi}{2}$ with the line whose direction coefficient are (l, m), then prove that **8**

$$l' = -\frac{1}{H}(Fl + Gm),\ m' = \frac{1}{H}(El + Fm)$$

OR

c) Show that the direction of curves bisecting the angle between the orthogonal parametric curves are $\left(\pm\frac{1}{\sqrt{2}}\frac{1}{\sqrt{E}}, \pm\frac{1}{\sqrt{2}}\frac{1}{\sqrt{G}}\right)$. **8**

d) Show that the parameters on a surface can always be chosen so that the curves of the given family & the orthogonal trajectories become parametric curves. **8**

UNIT – II

2. a) Prove that the curves of family $\frac{v^3}{u^2} =$ constant are geodesics on a surface with the metric $v^2du^2 - 2uvdudv + 2u^2dv^2$, $u > 0,\ v > 0$. **8**

b) Show that with S is a parameter, the component of the geodesic curvature vector are given by **8**

$$\lambda = \frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial V'} = -\frac{1}{H^2}\frac{V}{u'}\frac{\partial T}{\partial v'} \text{ and}$$

$$\mu = \frac{1}{H^2}\frac{V}{v'}\frac{\partial T}{\partial u'} = -\frac{1}{H^2}\frac{U}{v'}\frac{\partial T}{\partial u'}$$

OR

c) When a geodesic makes an angle θ with the curves v = constant of a geodesic coordinate system for which $ds^2 = du^2 + Gdv^2$, then prove that $\frac{d\theta}{ds} + \frac{\partial}{\partial u}\left(\sqrt{G}\right) v' = 0$. **8**

d) Find the Gaussian curvature at any point of the sphere with representation $r = a\ (\sin u\ \cos v,\ \sin u\ \sin v,\ \cos u)$ where $0 < u < \pi,\ 0 \le v < 2\pi$. **8**

UNIT – III

3. a) Show that the principal direction are given by **8**

$\left(EM-FL\right)l^2+(EN-GL)lm+(FN-GM)m^2=0$.

b) Find the normal curvature of the right helicoid $r(u,v)=(u\ \cos v,\ u\sin v, cv)$ at a point on it. **8**

OR

c) Show that a necessary & sufficient condition that the lines of curvature be parametric curves is that F = 0, M = 0. **8**

d) Prove that every space curve is a geodesic on its rectifying developable. **8**

UNIT – IV

4. a) If N is the surface normal, then prove that $N_1 \times N_2 = \dfrac{LN-M^2}{H}\ N$. **8**

b) Prove that the parallel surface of a minimal surface are surface for which **8**

$R_a + R_b =$constant, where $R_a = \dfrac{1}{K_a}$ and $R_b = \dfrac{1}{K_b}$.

OR

c) If $\overline{K}$ and $\overline{\mu}$ are the Gaussian curvature and mean curvature of $\overline{S}$, then prove that **8**

$\overline{K}=\dfrac{Ke}{(1+2\mu a+Ka^2)}$. $\overline{\mu}=\dfrac{(\mu+aK)e}{1+2\mu a+Ka^2}$ where $e=\pm 1$.

d) If $N_1=\dfrac{\partial N}{\partial u}$ and $N_2=\dfrac{\partial N}{\partial v}$, then prove that **8**

i) $N_1=\dfrac{1}{H^2}\left[(FM-GL)r_1+(FL-EM)r_2\right]$ and

ii) $N_2=\dfrac{1}{H^2}\left[(FN-GM)r_1+(FM-EN)r_2\right]$

5. a) Prove that if dS represents the element of area PQRS on the surface dS = H du dv. **4**

b) If a geodesic on a surface of revolution cuts the meridian at constant angle, then prove that the surface is a right cylinder. **4**

c) Find L, M, N for the sphere **4**
r = (a cosu cosv, a cosu sinv, a sinu)
where u is the latitude & v is the longitude.

d) From Weingarten equations, prove that $H\left[N,\ N_1, r_1\right]=EM-FL$. **4**

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